Digital Image Processing
15CS753

MODULE-2

Image Enhancement In The Spatial Domain

- Image enhancement using arithmetic and logical operators.
- Arithmetic operations are carried out between corresponding pixel pairs.
- The four arithmetic operations are denoted as

$$s(x, y) = f(x, y) + g(x, y)$$
$$d(x, y) = f(x, y) - g(x, y)$$
$$p(x, y) = f(x, y) \times g(x, y)$$
$$v(x, y) = f(x, y) \div g(x, y)$$

- These operations are performed between corresponding pixel pairs in f and g for x = 0, 1, 2… M-1 and y = 0, 1, 2… N-1 where, M and N are the row and column sizes of the images

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- Clearly, images s, d, p, and v are also of size M X N.
- Note that image arithmetic defined above involves images of the same size.
- Through the following examples we will see the important role played by arithmetic operations in digital image processing.
- Addition:
- Let g(x, y) denote a corrupted image formed by the addition of noise γ(x, y), to a noiseless image f(x, y); that is,

$$g(x, y) = f(x, y) + \eta(x, y)$$

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- Here we assume that at every pair of coordinates (x, y) the noise is uncorrelated and has zero average value.
- The following procedure is used to reduce the noise content by adding a set of noisy images, $\{g_i(x, y)\}$
- This is a technique used frequently for image enhancement..
- If the noise satisfies the constraints just stated, it can be shown that if an image $\bar{g}(x, y)$ is formed by averaging K different noisy images

$$\bar{g}(x, y) = \frac{1}{K} \sum_{i=1}^{K} g_i(x, y)$$

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- Then the expected value of g` is given by

$$E\{\overline{g}(x, y)\} = f(x, y)$$

- Variance is given by

$$\sigma^2_{\overline{g}(x,y)} = \frac{1}{K}\sigma^2_{\eta(x,y)}$$

- The standard deviation (square root of the variance) at any point in the average image is

$$\sigma_{\overline{g}(x,y)} = \frac{1}{\sqrt{K}}\sigma_{\eta(x,y)}$$

- As K increases, above equations indicate that the variability (as measured by the variance or the standard deviation) of the pixel values at each location (x, y) decreases.

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- Because $E\{\overline{g}(x, y)\} = f(x, y)$, this means that $\overline{g}(x, y)$ approaches f(x,y) as the number of noisy images used in the averaging process increases.
- An important application of image averaging is in the field of astronomy, where imaging under very low light levels frequently causes sensor noise to render single images virtually useless for analysis.

Imag

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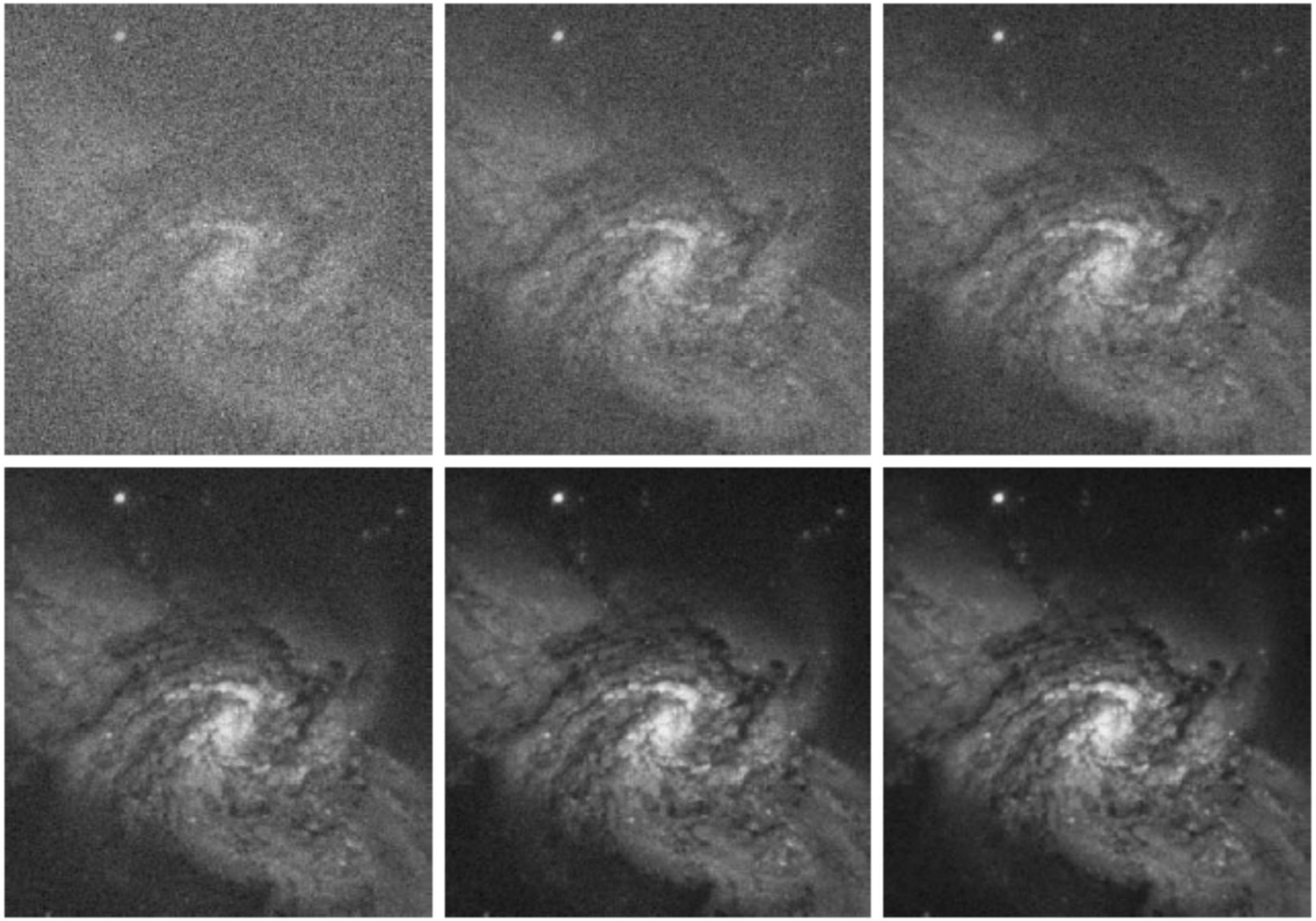

a b c
d e f

FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

Image Enhancement In The Spatial Domain

- Image Subtraction:
- An application of image subtraction is in the enhancement of differences between images.
- Consider the following figures.

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- Here second image is obtained by setting LS bit of every pixel of the first image to zero
- Visually both look alike
- Suppose if we subtract one image from the other, the differences would be clear as shown below

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- Black regions in the image are indicating those locations where there were no differences between two images
- One more use of image subtraction is in medical imaging called mask mode radiography.
- Consider image differences of the form.

$$g(x, y) = f(x, y) - h(x, y)$$

- In this case , the mask h(x, y), is an X-ray image of a region of a patient's body captured by an intensified TV camera (instead of traditional X-ray film) located opposite an X-ray source

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- The procedure consists of injecting an X-ray contrast medium into the patient's bloodstream, taking a series of images called live images [samples of which are denoted as f(x, y)] of the same anatomical region as h(x, y), and subtracting the mask from the series of incoming live images after injection of the contrast medium.
- The net effect of subtracting the mask from each sample live image is that the areas that are different between f(x, y) and h(x, y) appear in the output image, g(x, y), as enhanced detail.

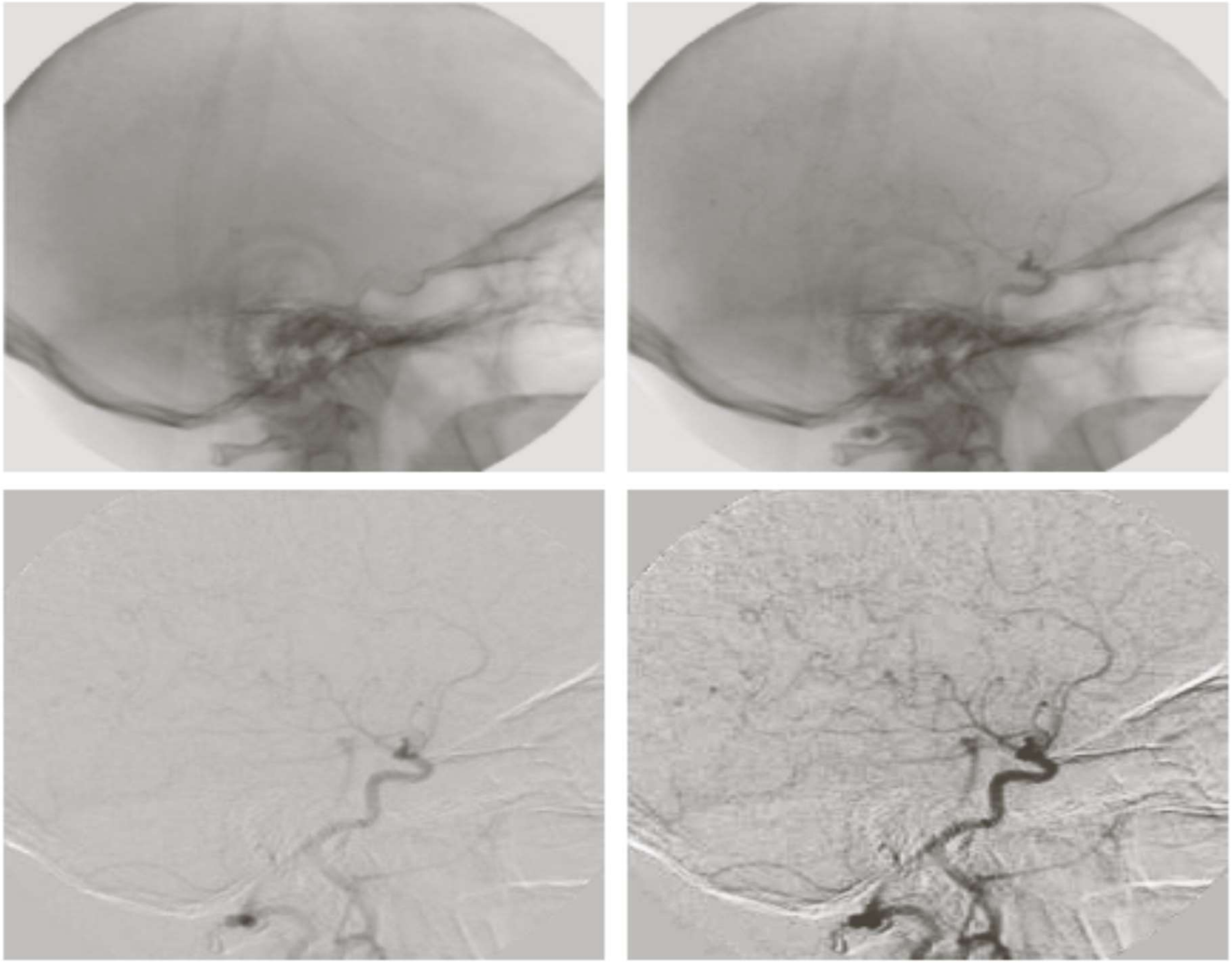

Image Enhancement In The Spatial Domain

- Figure (a) shows a mask X-ray image of the top of a patient's head prior to injection of an iodine medium into the bloodstream,
- Fig. (b) is a sample of a live image taken after the medium was injected.
- Figure (c) is the difference between (a) and (b).
- Some fine blood vessel structures are visible in this image.
- The difference is clear in Fig. (d), which was obtained by enhancing the contrast in (c).
- Figure (d) is a clear "map" of how the medium is propagating through the blood vessels in the subject's brain.

Image Enhancement In The Spatial Domain

- Image Multiplication (and Division):
- An important application of image multiplication (and division) is shading correction.
- Suppose that an imaging sensor produces images that can be modeled as the product of a "perfect image," denoted by $f(x, y)$, and a shading function, $h(x, y)$; that is $g(x, y) = f(x, y)\, h(x, y)$
- If $h(x, y)$ is known, we can obtain $f(x, y)$ by multiplying the sensed image by the inverse of $h(x, y)$(i.e., dividing g by h).
- If $h(x, y)$ is not known, but access to the imaging system is possible, we can obtain an approximation to the shading function by imaging a target of constant intensity.
- When the sensor is not available, we often can estimate the shading pattern directly from the image.
- Examples of shading correction are as shown in Figure

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a b c

FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Image Enhancement In The Spatial Domain

- Another common use of image multiplication is in masking, also called region of interest (ROI), operations.
- The process, consists of multiplying a given image by a mask image that has 1s in the ROI and 0s elsewhere.
- There can be more than one ROI in the mask image.
- The shape of the ROI can be arbitrary, although rectangular shapes are used frequently for ease of implementation.

a b c

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

Image Enhancement In The Spatial Domain

- Logical operations
- When dealing with binary images, we can think of foreground (1-valued) and background (0-valued) sets of pixels.
- Then, if we define regions (objects) as being composed of foreground pixels, the set operations like Union, Intersection and Compliment become operations between the coordinates of objects in a binary image.
- It is common practice to refer to union, intersection, and complement as the OR, AND, and NOT logical operations, in which 1 and 0 denote true and false, respectively..

Image Enhancement In The Spatial Domain

- Consider two regions (sets) A and B composed of foreground pixels.
- The OR of these two sets is the set of elements (coordinates) belonging either to A or B or to both.
- The AND operation is the set of elements that are common to A and B.
- The NOT operation of a set A is the set of elements not in A.
- Since we are dealing with images, if A is a given set of foreground pixels, NOT(A) is the set of all pixels in the image that are not in A
- We can think of this operation as turning all elements in A to 0 (black) and all the elements not in A to 1 (white)..
- These operations can be depicted as ..

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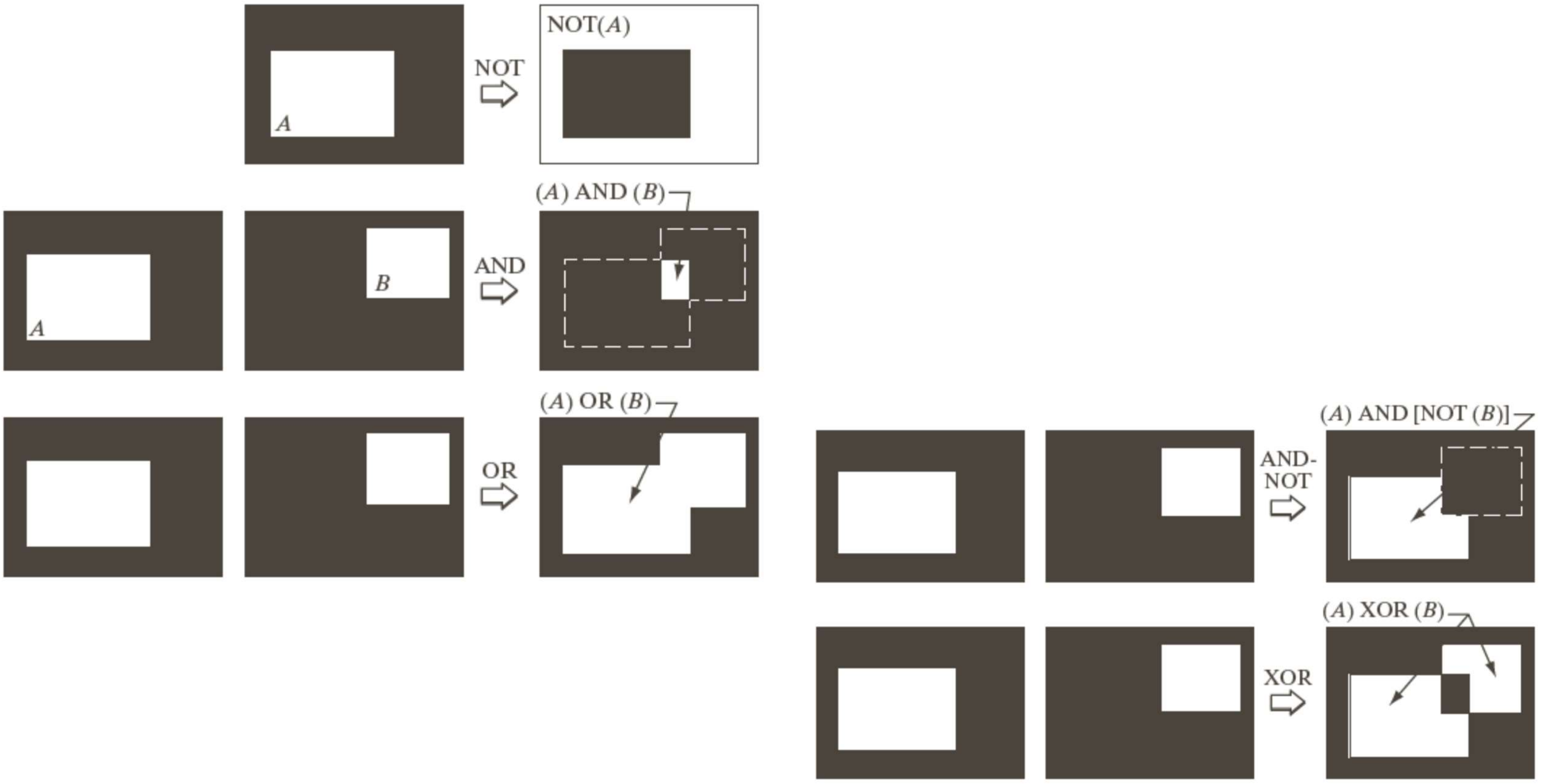

Image Enhancement In The Spatial Domain

- AND/ OR operations on gray scale images:
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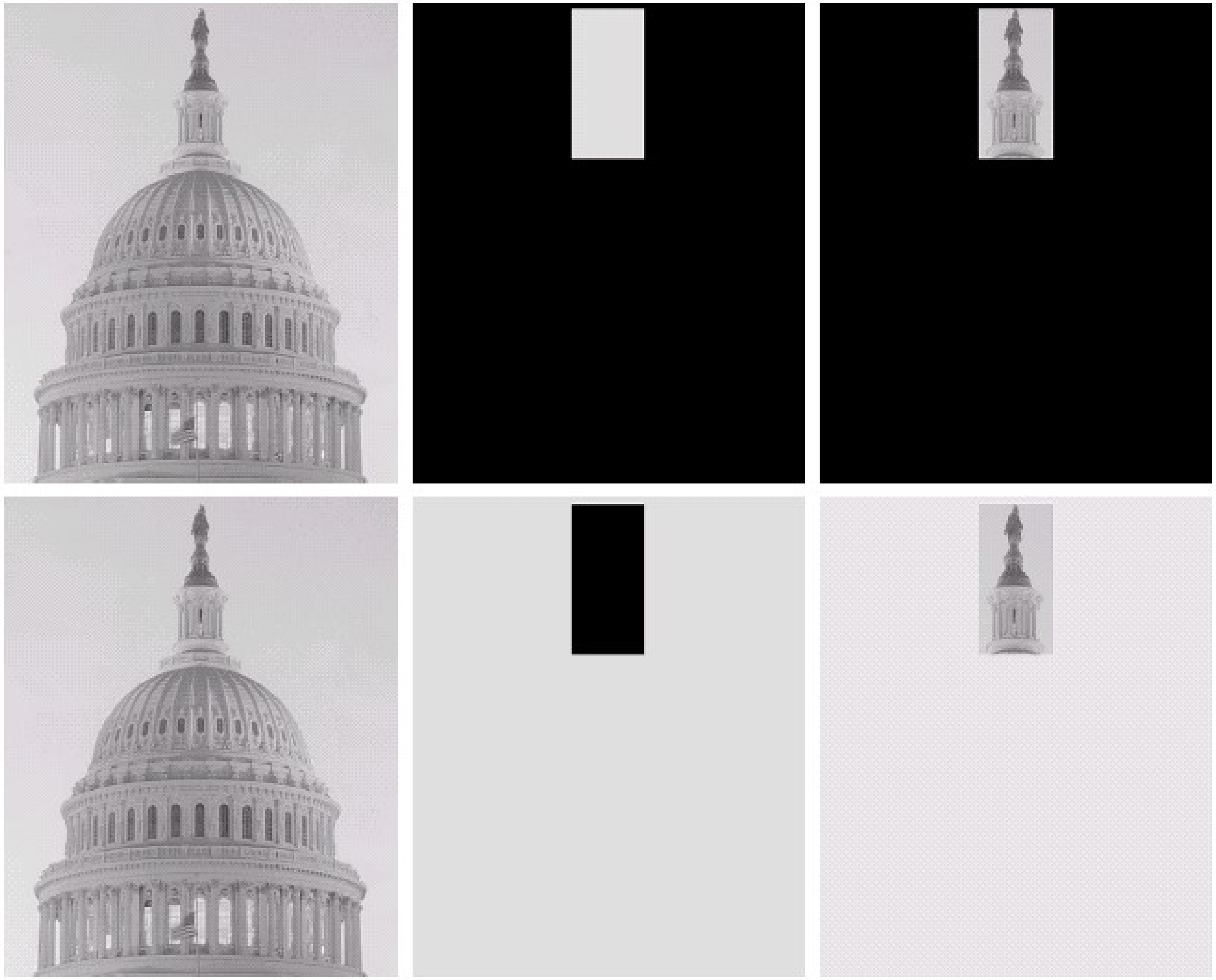

a	b	c
d	e	f

FIGURE 3.27 (a) Original image. (b) AND image mask. (c) Result of the AND operation on images (a) and (b). (d) Original image. (e) OR image mask. (f) Result of operation OR on images (d) and (e).

Image Enhancement In The Spatial Domain

- Basics of Spatial Filtering
- The term filter is taken from frequency domain in which filtering refers to accepting or rejecting certain frequency components
- E.g.: LPF, HPF
- Net effect produced in applying LPF is blurring (smoothing) of an image
- The same can be achieved by using spatial filters (spatial masks, kernels, templates, windows)
- Mechanics of Spatial filtering:
- Consider the image which we saw earlier showing the neighbourhood

Image Enhancement In The Spatial Domain

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Image Enhancement In The Spatial Domain

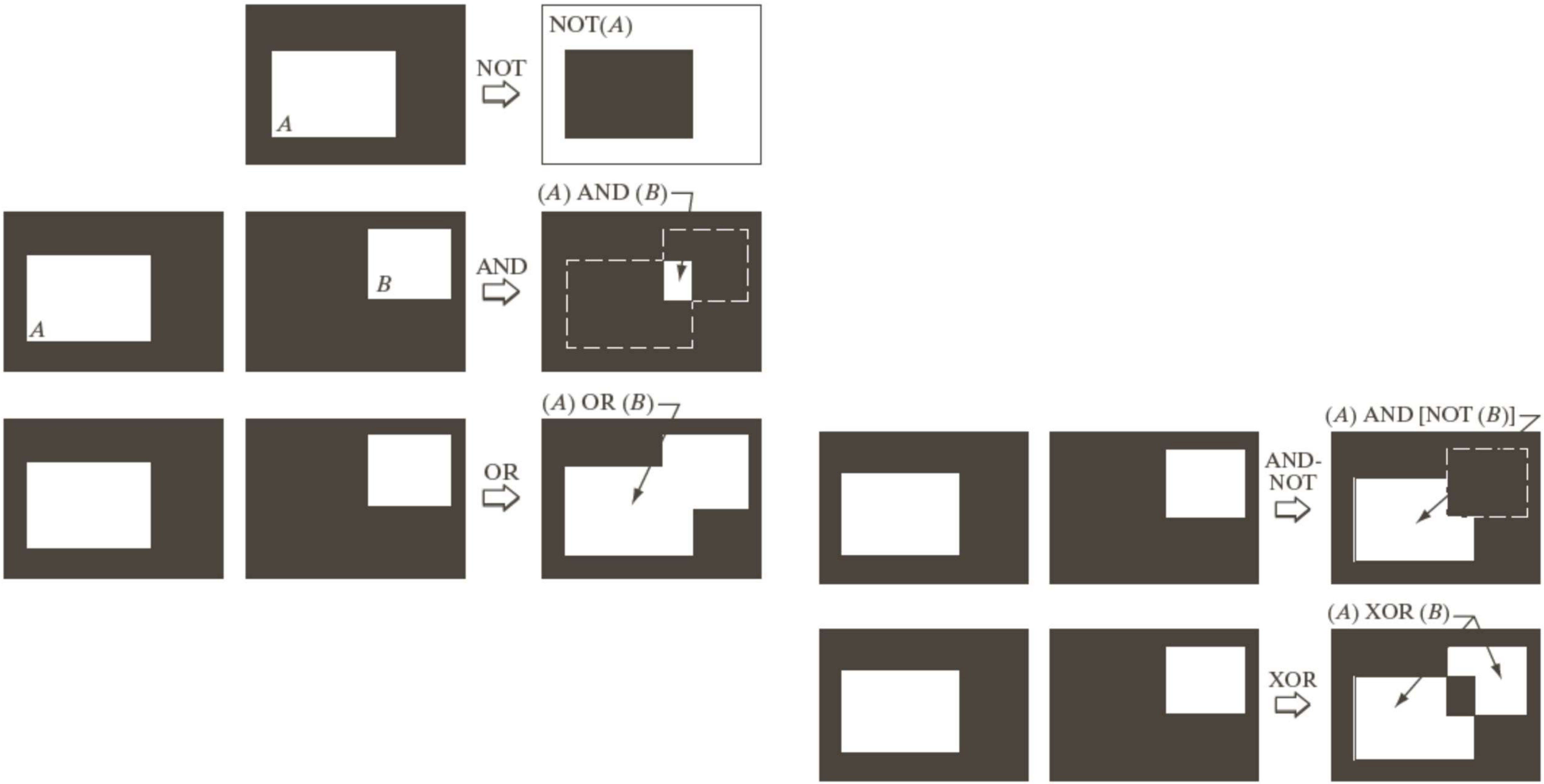

Image Enhancement In The Spatial Domain

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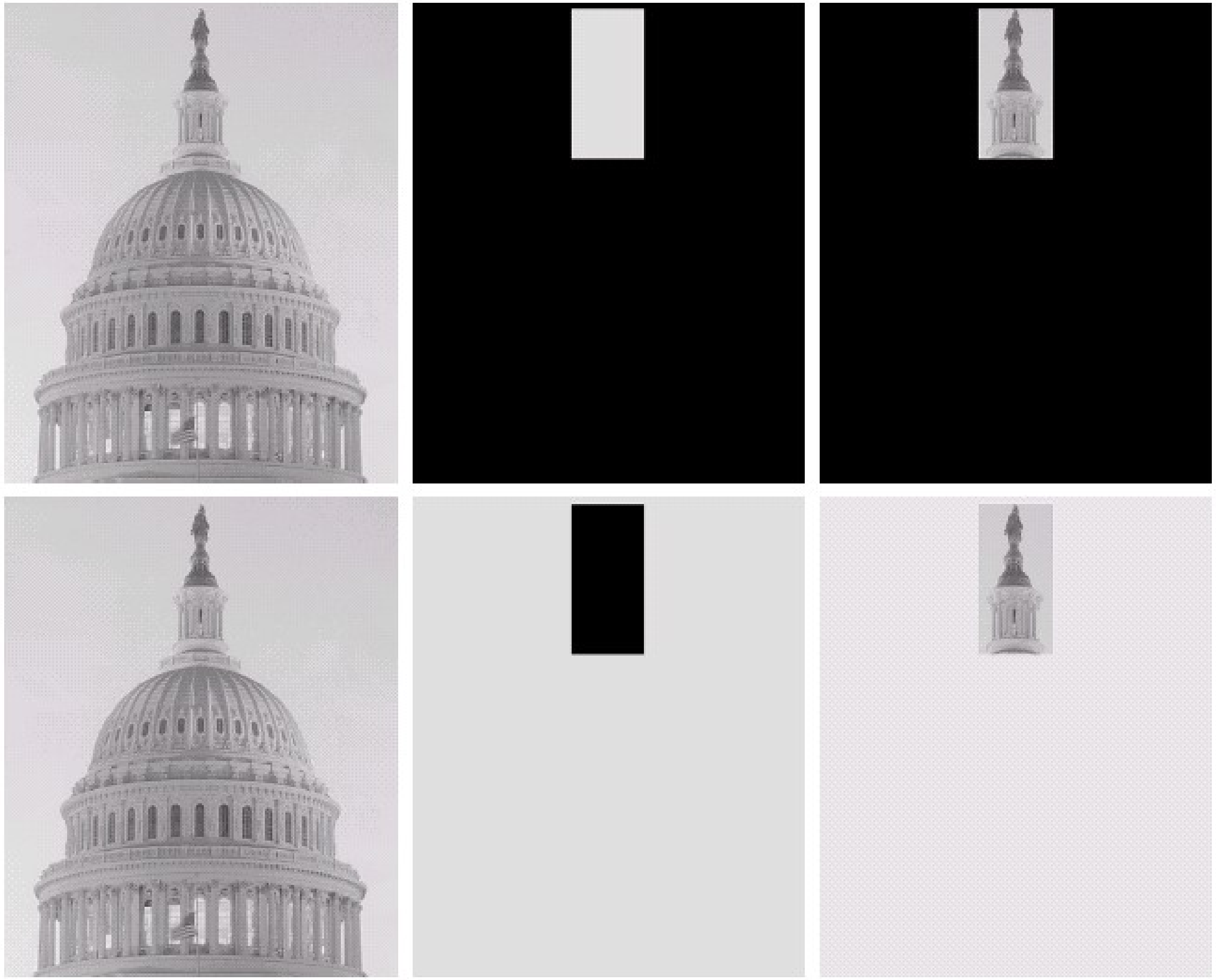

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Image Enhancement In The Spatial Domain

Padded *f*

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(b)

- Initial position of the filter mask to perform correlation is shown in the figure below

Initial position for ***w***									Fu
1	2	3	0	0	0	0	0	0	0
4	5	6	0	0	0	0	0	0	0
7	8	9	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

(c)

Image Enhancement In The Spatial Domain

Full correlation result

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	9	8	7	0	0	0
0	0	0	6	5	4	0	0	0
0	0	0	3	2	1	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(d)

Cropped correlation result

0	0	0	0	0
0	9	8	7	0
0	6	5	4	0
0	3	2	1	0
0	0	0	0	0

(e)

- We can see that the result is 180° rotated as compared to w function

Image Enhancement In The Spatial Domain

- For convolution, we pre-rotate the mask and repeat the sliding sum as seen earlier.
- The results will be as shown below

Rotated w

9	8	7	0	0	0	0	0	0
6	5	4	0	0	0	0	0	0
3	2	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(f)

Full convolution result

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	1	2	3	0	0	0
0	0	0	4	5	6	0	0	0
0	0	0	7	8	9	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(g)

Cropped convolution result

0	0	0	0	0
0	1	2	3	0
0	4	5	6	0
0	7	8	9	0
0	0	0	0	0

(h)

Image Enhancement In The Spatial Domain

- We can generalize the previous discussions into a mathematical equation
- The correlation of a filter w(x, y) of size m x n with an image f(x, y) denoted by w(x, y) * f(x, y) is given by

$$w(x,y) \star f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

- This expression is evaluated for all values of x and y so that, all elements of w visit every pixel in f.
- Similarly convolution of w(x, y) and f(x, y) is denoted by w(x, y)*f(x, y) and is given by

$$w(x,y) \bigstar f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

Image Enhancement In The Spatial Domain

- Here minus sign indicates that f is shifted by 180°.
- This is valid because correlation and convolution are commutative
- This equation is also evaluated for all values of displacement variables x and y so that every element of w visits every pixel of f.
- In all practical applications of algorithms first equation is used
- If correlation is needed w is used in the equation and if convolution is needed we use w shifted by 180°
- Now our interest is to know how convolution or correlation can be made use of for spatial filtering
- We often come across terms like convolution filter, convolution mask or convolution kernel.
- These terms are used to denote a spatial filter but not necessarily that the filter is used to perform true convolution

Image Enhancement In The Spatial Domain

- Similarly one more term we may meet often is convolving a mask with an image
- This simply implies the sliding sum-of-products operations discussed earlier and does not really differentiate between correlation and convolution

Image Enhancement In The Spatial Domain

- Generating spatial filter masks:
- Generating a *m* x *n* linear spatial filter means specifying the mn coefficients of the mask
- These are selected on the basis of what the filter is supposed to do
- Also we should remember that, what we can do with linear filtering is to perform sum of products
- E.g.:
- Suppose that we want to replace the pixels of an image by the average intensity of 3 x 3 neighborhood centered on those pixels.
- i.e. the average value at any location (x, y) in the image is the sum of nine intensity values in the 3 x 3 neighborhood centered on (x, y) divided by 9

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- Let z_i, for i = 1, 2, ..9 denote these intensities.
- Then the average is given by

$$R = \frac{1}{9}\sum_{i=1}^{9} z_i$$

- This is similar to the equation for R with coefficient values w_i being 1/9
- In other words, a linear filtering operation with 3 x 3 mask having coefficients 1/9 results in averaging operation
- We will see more such basic functions in the subsequent sections

- In some applications we may have a continuous function with two variables from which we need to obtain the spatial filter mask

Image Enhancement In The Spatial Domain

- Let us consider a Gaussian function whose general form is given by

$$h(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- where σ is the standard deviation, x and y are integers
- To generate a mask of 3 x 3 we need to sample it about its centre.
- i.e. w_1= h(-1,-1), w_2=h(-1, 0) … w_9=h(1, 1)
- An *m* x *n* filter mask is also generated in similar manner
- To generate a nonlinear filter we need specify the size of the neighbourhood and the operations to be performed on the image pixels contained in the neighbourhood.
- Nonlinear filters are more powerful and in some applications they can perform better than the linear filters.

Image Enhancement In The Spatial Domain

- Smoothing spatial filters:
- These filters are used for blurring and noise reduction of images
- Blurring is done usually in preprocessing tasks like removal of small details from an image before object extraction is done or in bridging the small gaps in lines or curves
- Noise reduction can be done by blurring with the linear filter or nonlinear filters
- Smoothing linear filters:
- The output of a smoothing linear spatial filter is average of the pixels contained in the neighborhood of the filter mask
- These are also called as averaging filters or low pass filters

Image Enhancement In The Spatial Domain

- The idea here is very simple
- Replace the value of every pixel in an image by the average of the intensity levels in the neighborhood defined by the filter mask
- The result of filtering is an image with reduced sharp transitions in intensities
- Usually noise contains sharp transitions in intensity levels.
- Hence smoothing can be used to reduce the noise in images

- What will happen at edges??
- Usually edges are characterized by sharp intensity transitions
- When averaging is applied to an image, even the edges get blurred which is undesirable.

Image Enhancement In The Spatial Domain

- Smoothing filters can also be used to smoothen the false contours that result from using an insufficient number of intensity levels
- The major use of this filter is in reduction of irrelevant details in an image
- Irrelevant means the pixel regions which are small in size as compared to the size of the filter mask
- E.g.: consider two filter masks as shown below

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

Image Enhancement In The Spatial Domain

- The first filter results in standard average of the pixels under the mask
- By substituting the values of w_i in the equation for R we get

$$R = \frac{1}{9}\sum_{i=1}^{9} z_i$$

- which is the average of the intensities of the pixel in its 3 x 3 neighborhood defined by the mask
- Instead of having coefficients as 1/9, here coefficients are 1.
- This is because, computation with coefficient 1 is efficient
- At the end of the filtering, the entire image is divided by 9

Image Enhancement In The Spatial Domain

- A spatial averaging filter in which all coefficients are equal is called as box filter
- Then second mask shown below gives weighted average

$$\frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

- Pixels are multiplied with different coefficients thereby giving more importance (weightage) for some pixels
- In this mask, the pixel at the center of the mask is multiplied by a higher value than others thereby giving this pixel more importance while calculating the average .

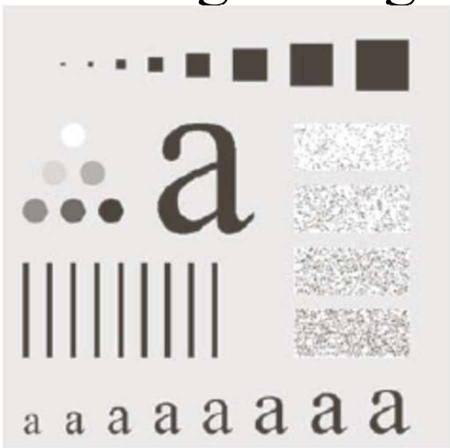
Image Enhancement In The Spatial Domain

- Other pixels are inversely weighted based on the distance from center
- i.e. diagonal elements are at far place as compared to immediate neighbors and hence are given with less weightage
- The idea behind such distance based weight is to reduce the blurring during filtering process
- The general implementation for filtering an M X N image with weighted averaging filter of size m x n is given by

$$g(x, y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t)}$$

Image Enhancement In The Spatial Domain

- The complete filtered image is obtained by applying above equation for $x = 0, 1, 2\ldots M-1$ and for $y = 0, 1, 2.. N-1$
- The denominator is sum of mask coefficients which is computed only once
- Let us analyze the effect of smoothing through the following figure



- This is an image of 500 x 500 size .
- For this image let us apply square averaging filters of size m = 3, 5, 9, 15 and 35

Image E

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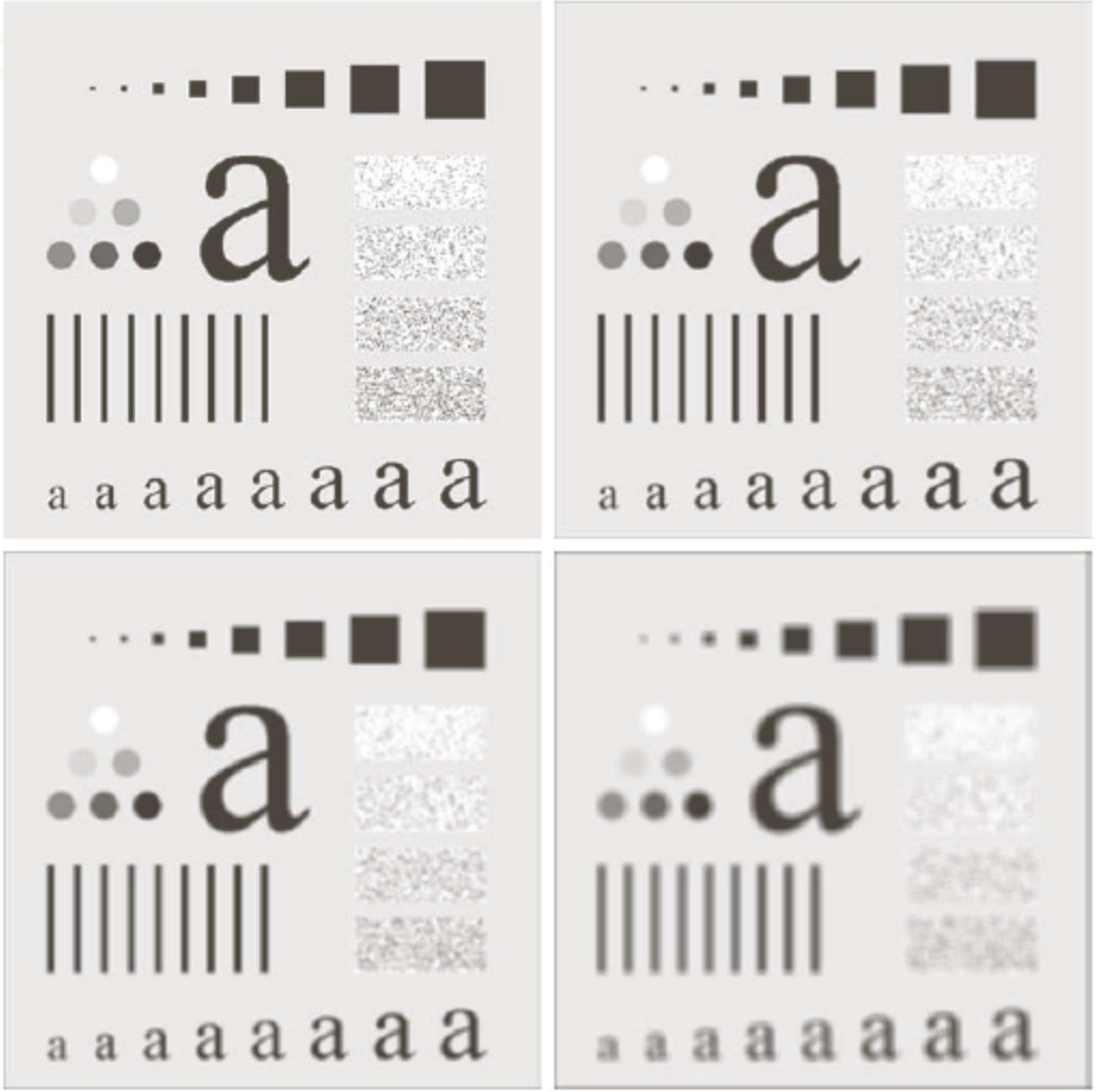

Image Enhancement In The Spatial Domain

- For m = 3,
- there is slight blur in the image.
- Details that are of same size as that of mask are affected more
- E.g. 3 x 3 and 5 x 5 black squares, small letter a and fine grain noise show significant blurring
- For m=5
- Result somewhat similar to m=3 with still more increase in blurring
- For m= 9
- Considerably more blurring.
- Black circle is not clearly distinct as compared to the previous images showing the blending effect in which blurring has an object whose intensities are close to that of its neighboring pixels

Image E

- .

Image Enhancement In The Spatial Domain

- For m= 15 and 35 results are severely blurred images
- This type of blurring can be used to remove small objects from the image
- E.g. three small squares, two circles and most of the noise are blended into the background

- Thus the important application of spatial averaging is to blur the image for getting the gross representation of the object of interest
- While achieving this, the intensity of small objects blend with the background and the larger objects become easy to detect
- Let us take example of Hubble telescope image

Image Enhancement In The Spatial Domain

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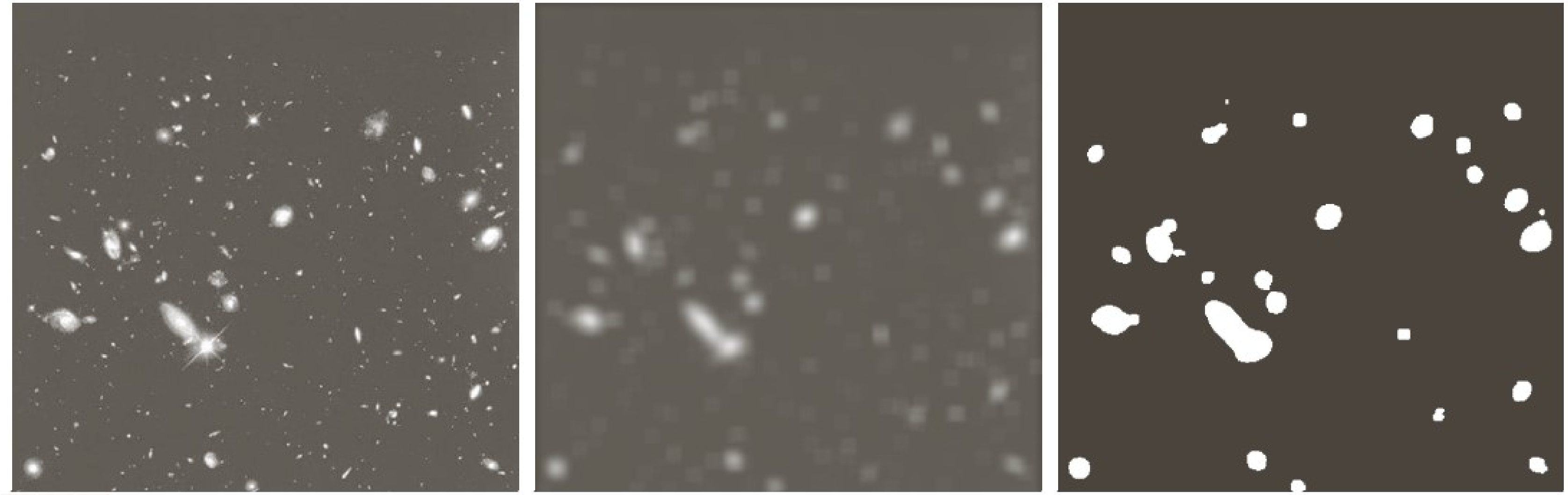

FIGURE 3.34 (a) Image of size 528 × 485 pixels from the Hubble Space Telescope. (b) Image filtered with a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

- figure c is obtained by applying a threshold function with threshold value tc = 25% of the highest intensity in the blurred image
- When a and c are compared we can see that c contains the representation of largest and brightest objects in the image a

Image Enhancement In The Spatial Domain

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Image Enhancement In The Spatial Domain

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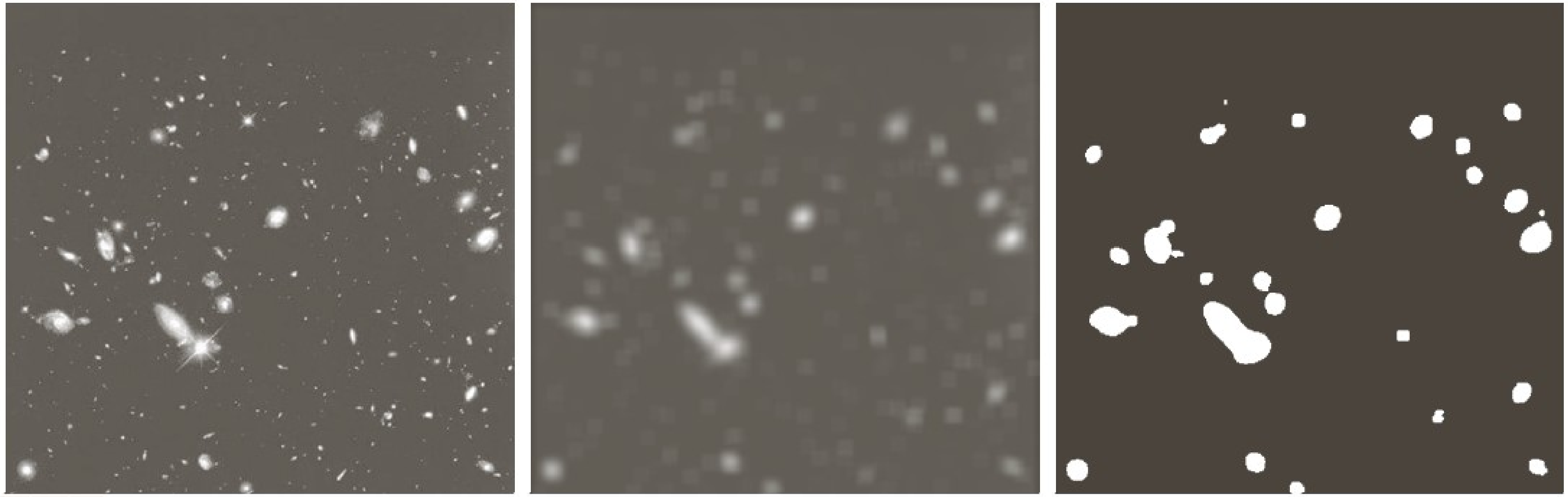

a b c

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Image Enhancement In The Spatial Domain

- Order Static (Non-Linear) Filters
- These filters are nonlinear filters whose response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter mask and then replacing the value of the center pixel with the value determined by the ranking result
- Well known filter in this category is median filter
- As the name says, it replaces the pixel value by the median of the intensities in the neighborhood of that pixel.
- These filters are popular as they provide excellent noise reduction for some types of random noises with considerably less blurring as compared to linear filters.
- E.g.: salt and pepper noise

Image Enhancement In The Spatial Domain

Image Enhancement In The Spatial Domain

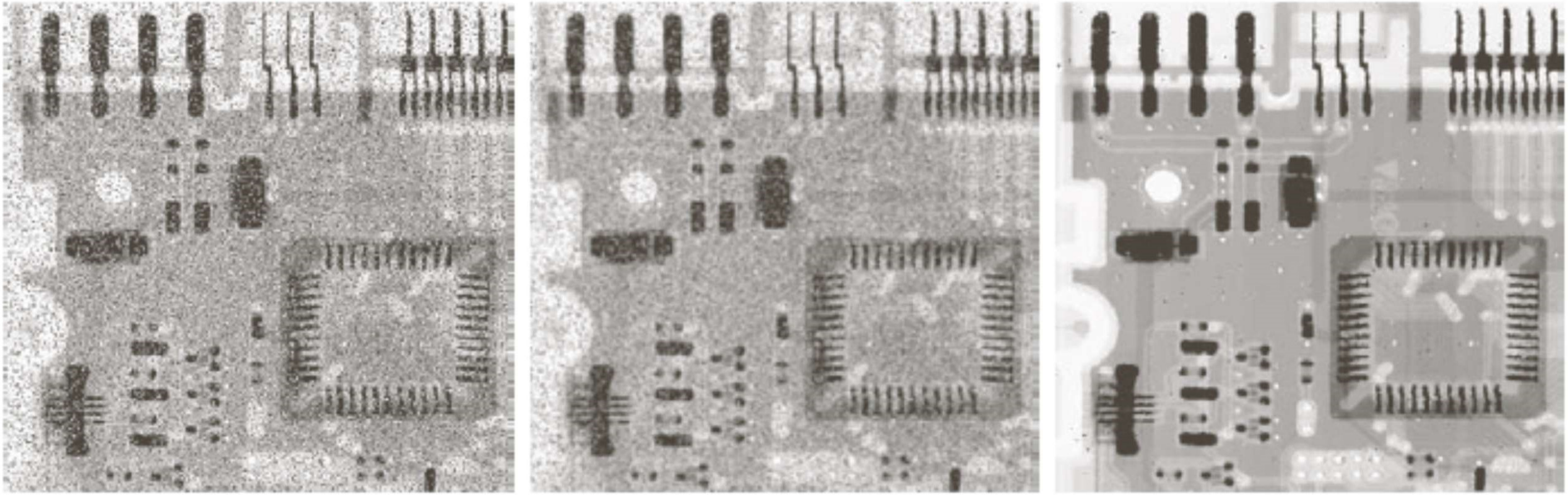
Image Enhancement In The Spatial Domain

- The median ξ of a set of values is such that, half the values in the set are less than or equal to ξ and remaining are greater than or equal to ξ.
- To perform a median filtering at any point in the image, we first sort the values of the pixel in the neighborhood, determine the median and then assign that value to the corresponding pixel in the output image
- E.g. in 3 X 3 mask median is 5^{th} largest value, in 5 X 5 it is 13^{th} largest value and so on..
- In case if several values in the neighborhood are equal, then all equal values are grouped
- E.g. in a 3 X 3 neighborhood, if the values are 10, 20, 20, 20, 15, 20, 20, 25, 100 then these are sorted as
- 10, 15, 20, 20, 20, 20, 20, 25, 100

Image Enhancement In The Spatial Domain

- This results in median value of 20
- The principal idea of median filter is to force points with distinct intensities to be more like their neighbors
- There are other filters also under nonlinear filters namely max, min etc.
- .E.g.:

Image Enhancement In The Spatial Domain



a b c

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Image Enhancement In The Spatial Domain

- Sharpening spatial filters:
- Objective here to highlight the transitions in intensities
- Application of image sharpening electronic printing and medical imaging to industrial inspection and autonomous guidance in military systems
- We have seen that, blurring is done by pixel averaging in the neighborhood
- Averaging is somewhat similar to integration operation
- Thus we can say that, sharpening can be achieved by spatial derivation
- Here we study different ways of defining and implementing operators for sharpening by digital differentiation

Image Enhancement In The Spatial Domain

- The power of response of a derivative operator is proportional to the degree of discontinuity of the image at the point at which the operator is applied
- Thus image differentiation enhances edges and other discontinuities and deemphasizes areas with slow varying intensities
- .

Image Enhancement In The Spatial Domain

- Derivative of a digital function:
- Derivative of a digital function is expressed in terms of differences
- There are many ways to define differences
- But for us, any definition of first derivative
  - Must be zero in areas of constant intensity
  - Must be nonzero at the beginning of an intensity step or ramp and
  - Must be nonzero along ramps
- Any definition of second derivative
  - Must be zero at constant intensities
  - Must be nonzero at the beginning and end of an intensity step or ramp
  - Must be zero along ramps of constant slopes
- As we are dealing with digital quantities whose values are finite, the maximum possible intensity change is also finite

Image Enhancement In The Spatial Domain

- The shortest distance over which that change can occur is **between adjacent pixels**
- Basic form of a first order derivative of a one dimension function f(x) is

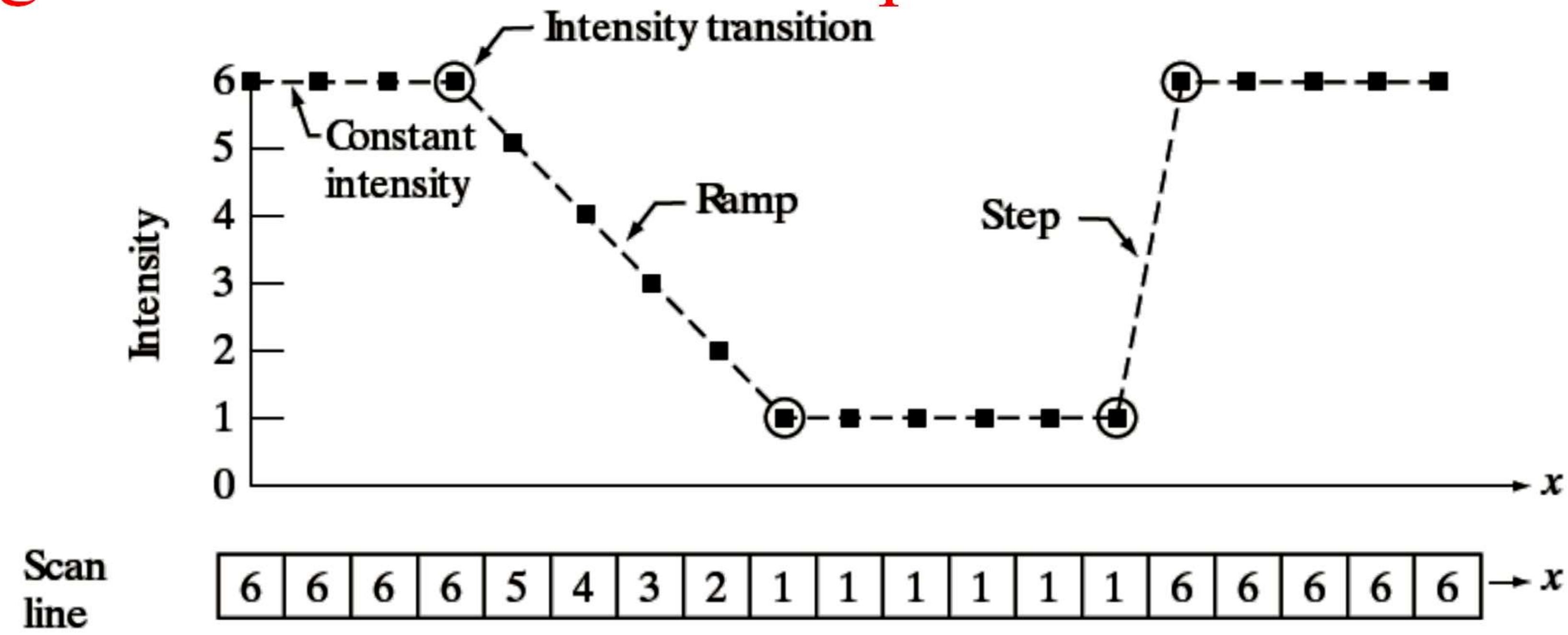
$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- Partial derivative is used here just to keep the notation same for both first order and second order derivative
- Similarly second order derivative is given by

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

- To understand similarities between first and second order derivatives let us take an example as shown

Image Enhancement In The Spatial Domain



- Center of the figure shows scan line or intensity profile of the image
- The values here are the intensity values in the scan line, which are plotted as black dots in the above image
- Dashed line connecting the dots helps as aid for visualization

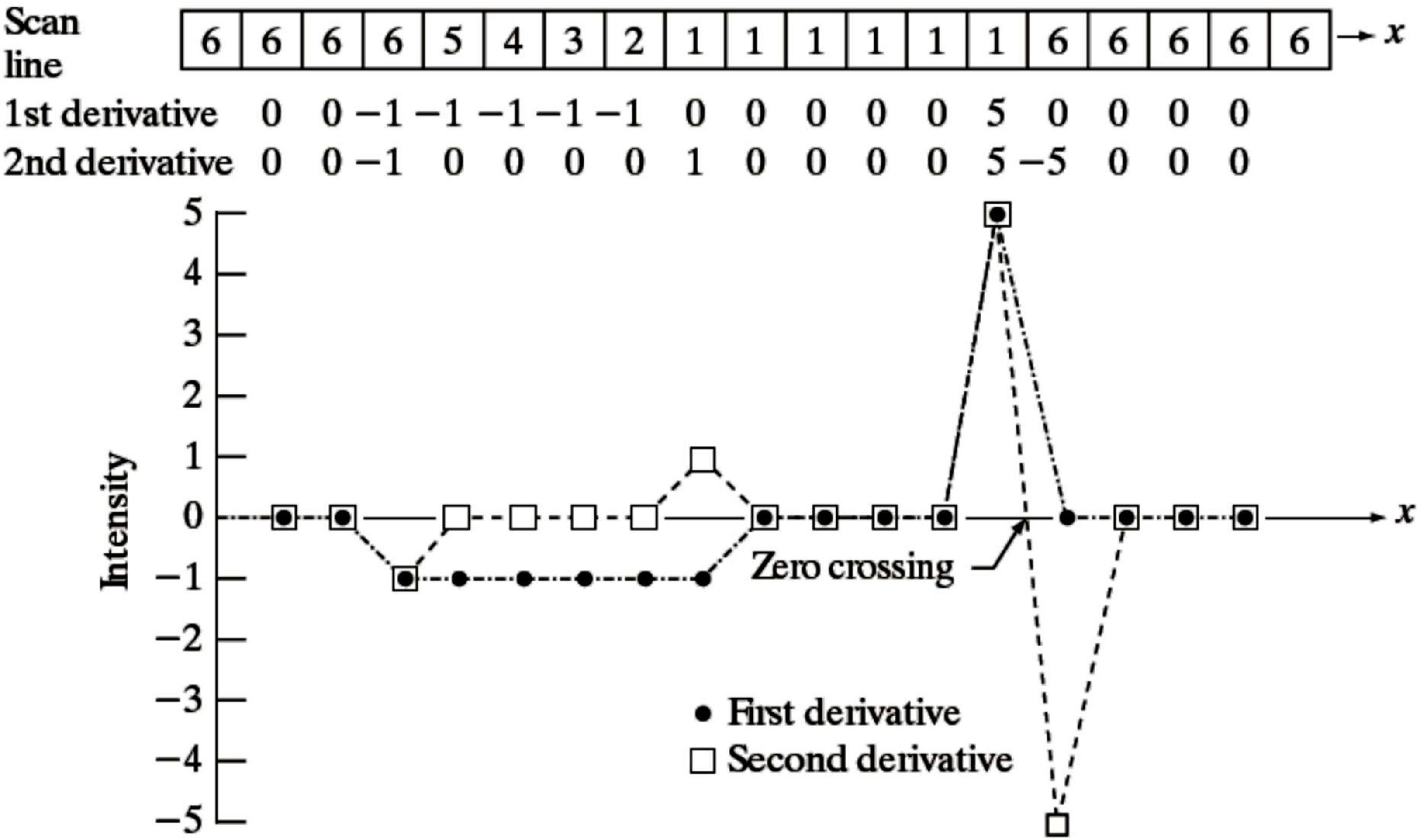
Image Enhancement In The Spatial Domain

- Scan line contains
  - intensity ramp, three sections of constant intensity and an intensity step
- The circles represent the beginning or end of intensity transitions
- To compute the first derivative at any location x, we subtract the value of the function at that location from the next location
- To compute the second order derivative , we use previous and next points
- These are computed using the previous two equaitons
- The results are as shown below

Scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6 → x
1st derivative		0	0	−1	−1	−1	−1	−1	0	0	0	0	0	5	0	0	0	0	
2nd derivative		0	0	−1	0	0	0	0	1	0	0	0	0	5	−5	0	0	0	

Image Enhancement In The Spatial Domain

- The plot of first and second order derivatives are shown in figure below



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- As we go from left to right of the intensity profile,
- We see there is an area of constant intensity, where derivatives are zero – condition 1 is satisfied
- Next we see an intensity ramp followed by a step – here at the beginning of the ramp and step, first order derivative is nonzero.
- Also second order derivative is nonzero at the beginning and end of both ramp and step – satisfies condition 2
- We can also see that, first derivative is nonzero and second derivative is zero along the ramp – condition 3 is satisfied

Image Enhancement In The Spatial Domain

- Note that the second derivative is changing sign at the beginning and end of the ramp and step
- Line joining these transitions in step, crosses the horizontal axis. This zero crossing property is used in edge detection
- Second derivative for Image Sharpening
- Here we consider implementation of 2-D second order derivatives and their use in image sharpening
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- $$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \quad \ldots\ldots 1$$
- It is known that, derivative of any order is linear hence Laplacian is a linear operator.
- This can be expressed in discrete form we use the basic definition seen earlier keeping in mind that we have to carry second variable too
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Image Enhancement In The Spatial Domain

- Thus in x direction we have
-
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- Similarly for y direction this is given by
- $$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y) \quad \ldots 3$$

- Thus from the last three equations we can write Laplacian operator as

$$\nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

Image Enhancement In The Spatial Domain

- This equation can be implemented using the filter mask as shown below

0	1	0
1	−4	1
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- This gives an isotropic result for rotations of 90°.
- The mechanics of implementation of this is similar to that of linear smoothing filters but with different coefficients
- We can add diagonal directions in the definition of digital Laplacian by adding two more components to the previous equation

Image Enhancement In The Spatial Domain

- The form of two new terms is same as in equations 2 or 3 but coordinates are along diagonal
- Since each diagonal term also contains a *-2f(x, y)* term, total subtracted from the differences would be *-8f(x, y)*
- The filter mask for this is shown below

1	1	1
1	−8	1
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- This mask gives isotropic results in increments of 45°

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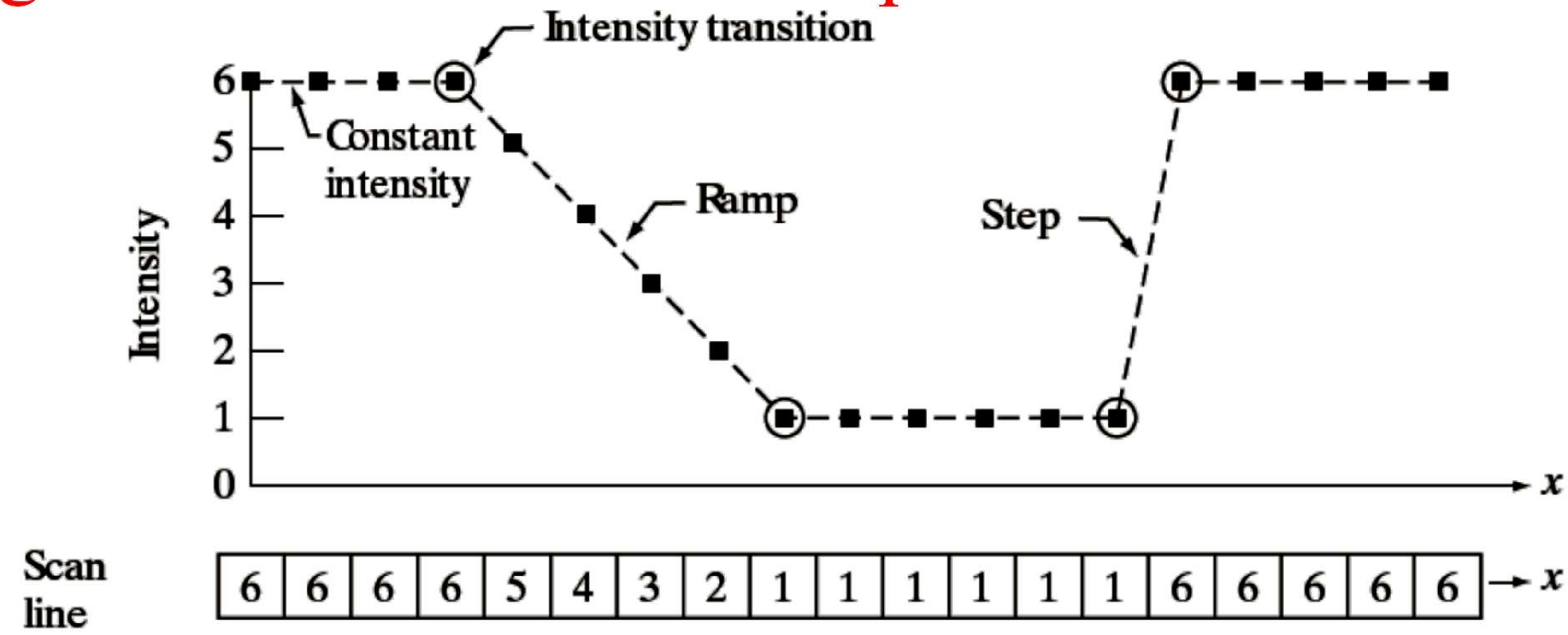

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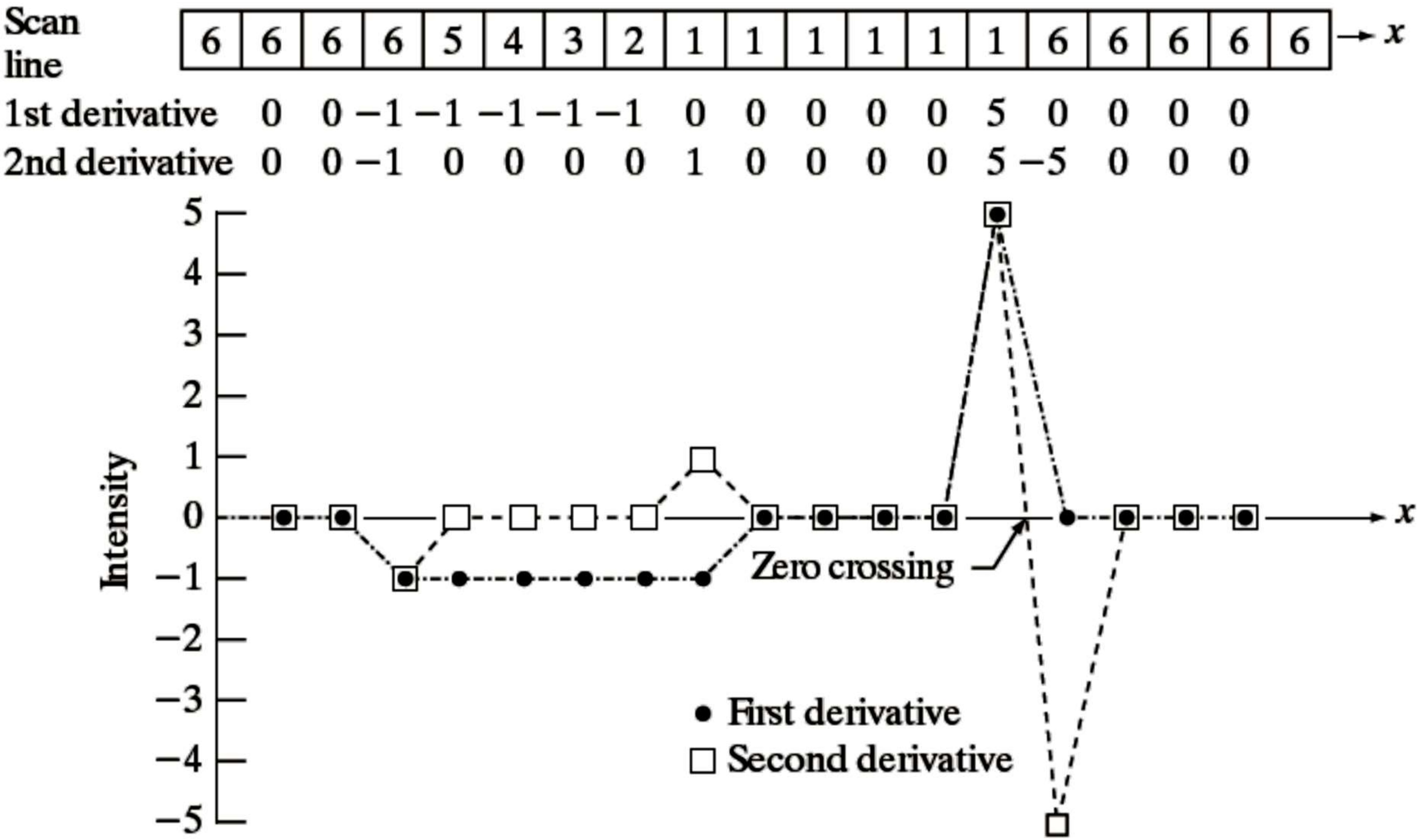

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−1	−1	−1
−1	8	−1
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- These are obtained from the definitions of second derivatives and are negatives of the last two masks
- Though these give equivalent results, we should keep in our mind about the difference in sign while combining a Laplacian filtered image with another image.

Image Enhancement In The Spatial Domain

- Since Laplacian is a derivative operator, it highlights intensity discontinuities in an image an deemphasizes regions with slow intensity variations
- This tends to produce the image that has grayish edge lines and other discontinuities superimposed on a dark featureless background
- Background features can be recovered while preserving the sharping effect of Laplacian by adding the Laplacian image to original image
- Note that, if the center of the Laplacian definition used has negative center coefficient then subtract the Laplacian image from original else go for addition
- Thus the basic way of using Laplacian for an image sharpening is given by .

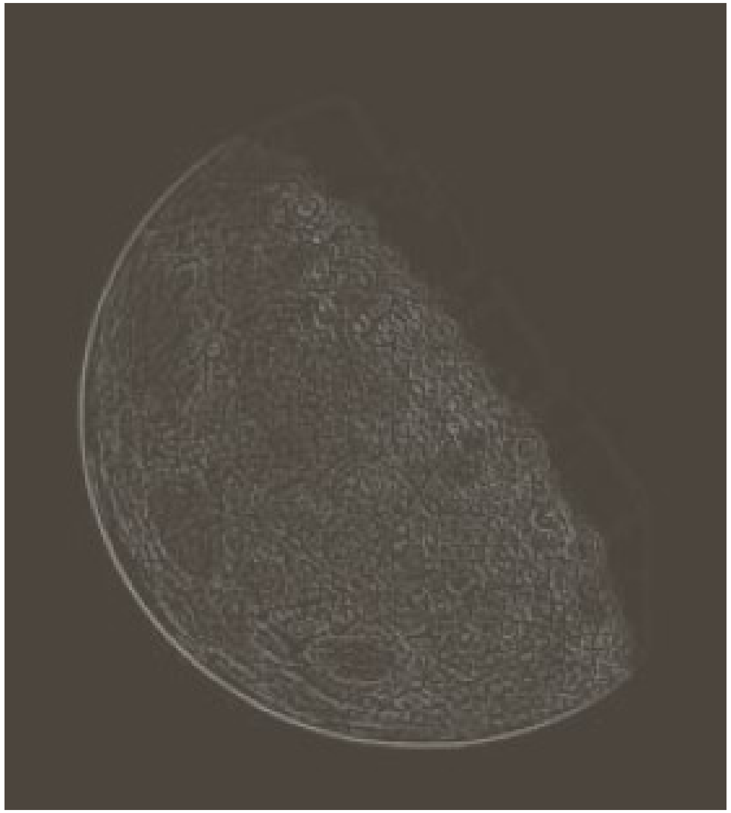
$$g(x, y) = f(x, y) + c\left[\nabla^2 f(x, y)\right]$$

Image Enhancement In The Spatial Domain

- Here f(x, y) and g(x, y) are original and sharpened image
- The constant c = -1 if the center coefficient of the mask is negative else c=1
- E.g.:
- Let us take up a blurred image of north pole as shown below

Image Enhancement In The Spatial Domain

- The result of filtering this with Laplacian mask with negative coefficient at the center is shown below



- Larger section of this image is black .
- This is because, Laplacian contains both positive and negative values and all negative values are rounded off to 0 while displaying

Image Enhancement In The Spatial Domain

- Now the image is subjected to a scaling operation
- Here the background is changed to grayish from black due to scaling..
- This is typical nature of Laplacian mask.
- This image is added with original image using the equation

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- with c = -1
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Image Enhancement In The Spatial Domain

Image Enhancement In The Spatial Domain

- Adding original image with Laplacian, restored the overall intensity variations in the image in which, Laplacian increased the contrasts at locations intensity discontinuities
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Image Enhancement In The Spatial Domain

- While comparing the results of Laplacian mask without and with diagonal coefficients we notice that the latter gives improved results in terms of image sharpness

- Unsharp masking and Highboost filtering
- This is a process used by many printing and publishing industry over many years
- The idea here is to subtract, unsharp (smoothened) image from the original image
- Steps involved :
- 1. blur the original image
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- Let $\bar{f}(x, y)$ denote blurred image
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$$g_{mask}(x, y) = f(x, y) - \bar{f}(x, y)$$

- Then we add a weighted portion of the mask to original image
- $g(x, y) = f(x, y) + k * g_{mask}(x, y)$
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Image Enhancement In The Spatial Domain

- Consider the following figure
- Intensity profile shown in fig. a is the horizontal scan line
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- Comparing this with result of 3.36 we can see that this is similar to what we got using second derivative
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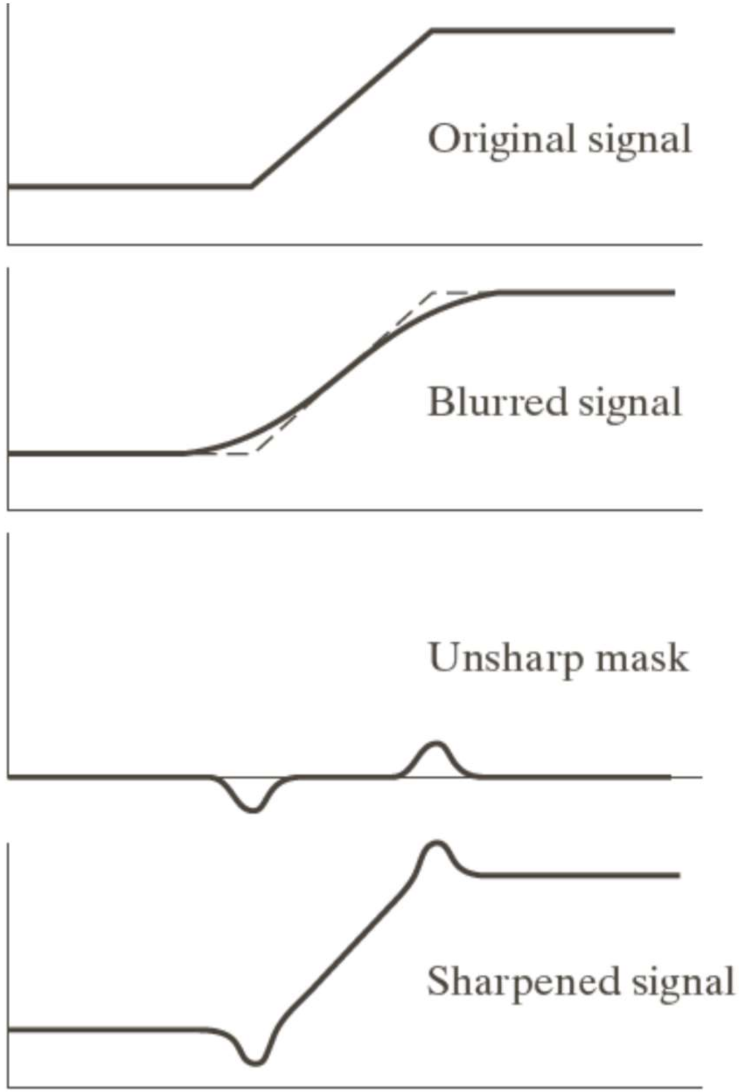

Image Enhancement In The Spatial Domain

- Last Figure is sharpened image obtained by adding the mask to original image
- The points at which change of slope in intensity occurs in the signal are now emphasized (sharpened)
-
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Ir… …ent In The Spatial Domain

- Fig 1 slightly blurred image
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- We can see that this image is slightly improved as compared to original image
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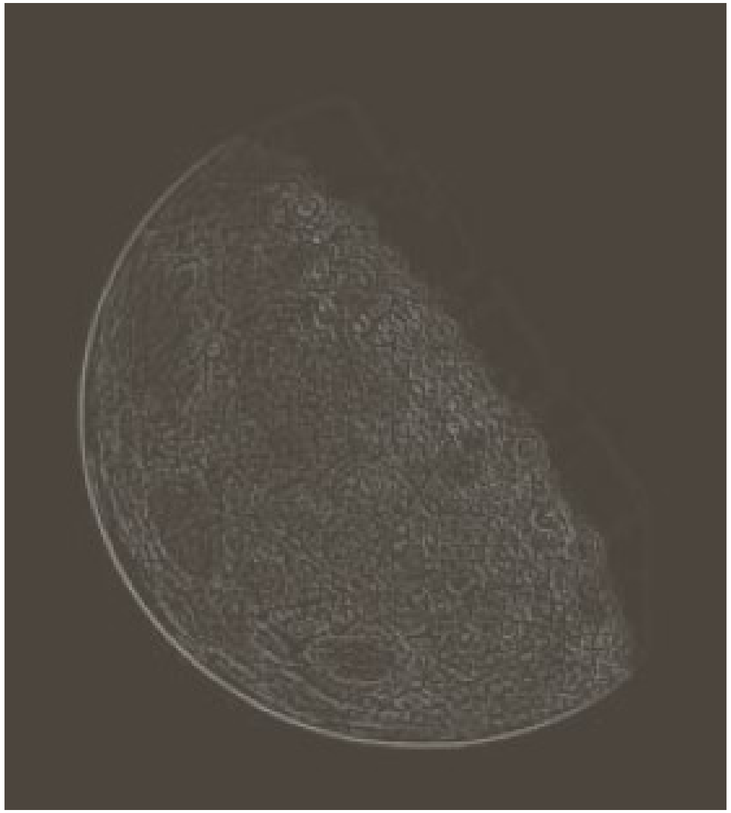

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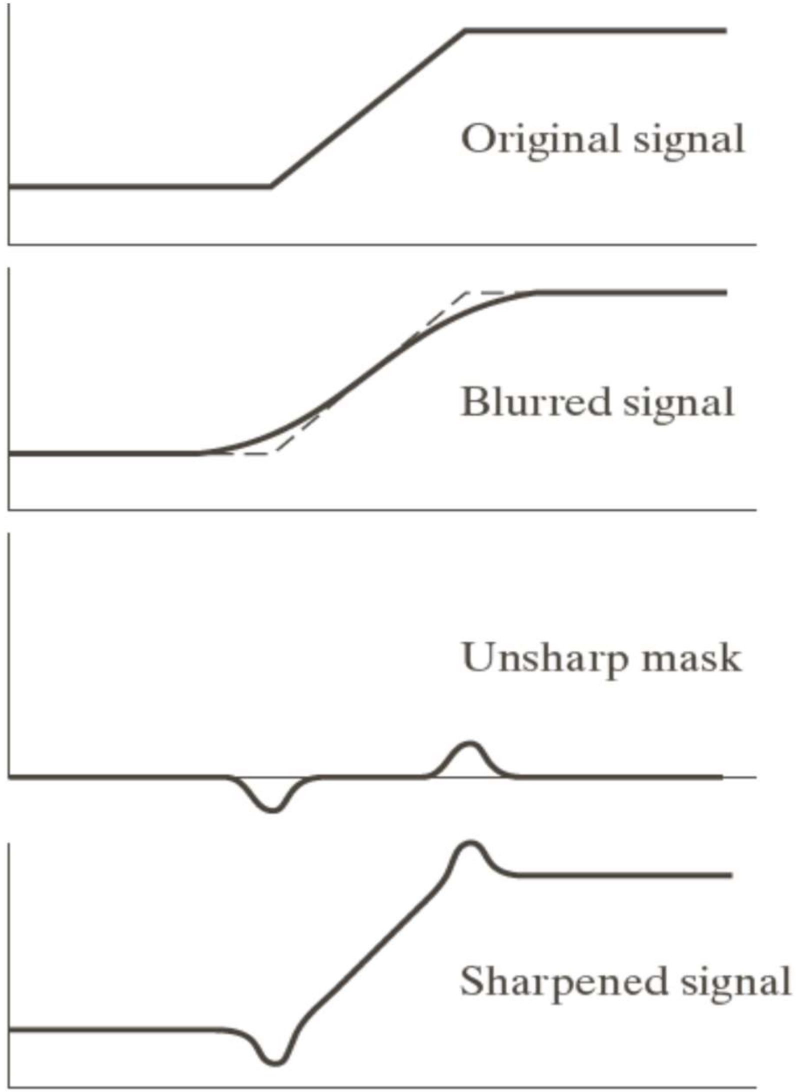

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Image Enhancement In The Spatial Domain

- Using first order derivatives for (non-linear) Image sharpening – The Gradient
- First derivatives in DIP are implemented using the magnitude of the gradient
- For a function *f(x, y)* the gradient of f at coordinates *(x, y)* is defined as the two-dimensional column vector

$$\nabla f \equiv \text{grad}(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

- This vector has a property that, it points in the direction of greatest rate of change of f at the location (x, y)
-

Image Enhancement In The Spatial Domain

- Magnitude of the vector is denoted by M(x, y) and is given by

$$M(x, y) = \text{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2}$$

- *M(x, y)* is a value at (x, y) of the rate of change in the direction of the gradient vector
- *M(x, y)* is an image of same size as the original, when *x* and *y* are allowed to vary over all pixel locations in *f*.
- This image is also known as gradient image or gradient
- In some implementations the above equation is approximated as .

$$M(x, y) \approx |g_x| + |g_y|$$

- This expression preserves the relative changes in intensity but isotropic property is lost

Image Enhancement In The Spatial Domain

- Note that, the isotropic properties of the discrete gradient are preserved only for a limited number of rotational increments that depend on filter mask used
- Most popular masks used to approximate gradient are isotropic at multiples of 90°.
- These results are independent whether we use first equation or gradient or second
- Now let us define discrete approximations to the previous two equations and from there formulate the filter masks
- To make discussions simple let us represent intensities of image in 3X3 region as shown in the figure

Image Enhancement In The Spatial Domain

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

- The center point z_5 denotes *f(x, y)* at an arbitrary location *(x, y)*
- z_1 denotes *f(x-1, y-1)* and so on..
- The simplest approximation of first order derivative seen earlier is $df/dx = f(x+1)-f(x)$ and $df/dy = f(y+1)-f(y)$.
- Using these we can write $g_x = z_8 - z_5$ and $g_y = z_6 - z_5$
- There are other definitions of first derivative too which are based on cross differences.

Image Enhancement In The Spatial Domain

- Using these we can write
- $g_x = (z_9 = z_5)$ and $gy = (z_8 - z_6)$. ---- (a)
- By using the first definition of gradient and equation (a) we may compute the gradient image as
- $$M(x, y) = \left[(z_9 - z_5)^2 + (z_8 - z_6)^2\right]^{1/2} \quad \text{---- (b)}$$
- By using the second definition of gradient and equation (a) we get
- $$M(x, y) \approx |z_9 - z_5| + |z_8 - z_6| \quad \text{---- (c)}$$
- Note that x and y has to be varied over the entire image
- Partial derivative terms needed in equation (a) can be implemented using two linear filter masks as shown in the figure

Image Enhancement In The Spatial Domain

−1	0
0	1

0	−1
1	0

- These masks are known as Roberts cross gradient operators
- Approximation to g_x and g_y using 3 X 3 neighborhood centered at z5 are given as

$$g_x = \frac{\partial f}{\partial x} = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$g_y = \frac{\partial f}{\partial y} = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

Image Enhancement In The Spatial Domain

- These equations can be implemented using the mask as shown below

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

- The difference between third and first row of 3 X 3 image region implemented by the mask on left side of the above image approximates the partial derivative in the x direction and other (difference between third and first column) gives the approximation of partial derivative on the y- direction.
- After computing partial derivatives with these masks we obtain the magnitude of the gradient

Image Enhancement In The Spatial Domain

- Substitution the values of gx and gy in the equation of gradient we get

$$M(x, y) \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

- These masks are called as Sobel operators
- The weight value of 2 for the center coefficient is to achieve some smoothing by giving more importance to the center point
- We can also see that, the coefficients in all masks seen here, when added would give sum as zero
- This means that, they would give a response of 0 in a area of constant intensity as a property of derivative. .

Image Enhancement In The Spatial Domain

- Note that computation of g_x and g_y are linear operation as they involve derivative and hence they can be implemented as a sum of product using spatial masks shown in the figure.
- Non linearity of sharpening with gradient comes into picture only with computation of $M(x, y)$ which involves squaring and square root or the absolute value
- These operations are performed after the linear process that results in g_x and g_y.
- E.g.
- Image of lense… .

Image Enhancement In The Spatial Domain

- .

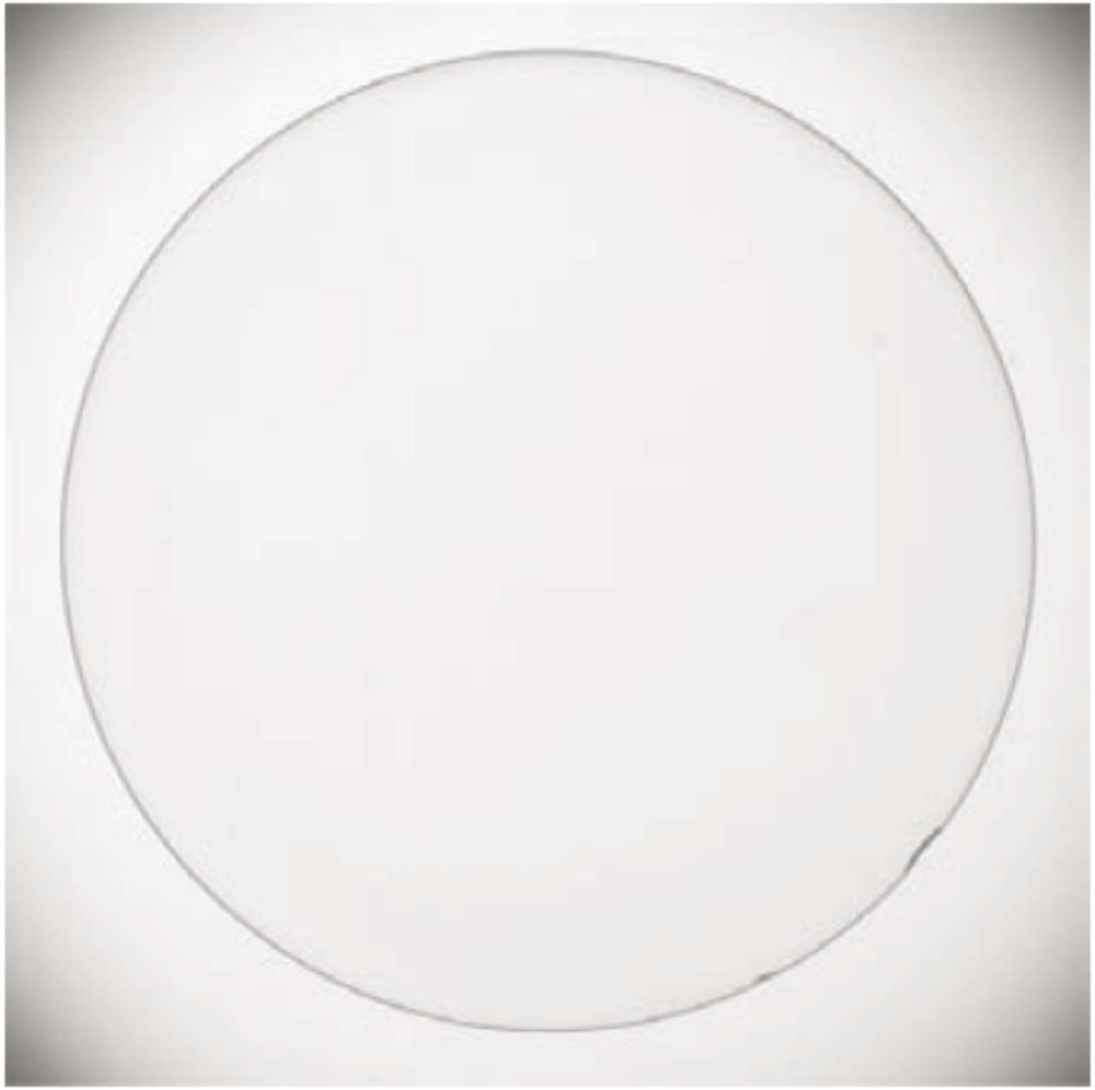


Image Enhancement In The Spatial Domain

- .

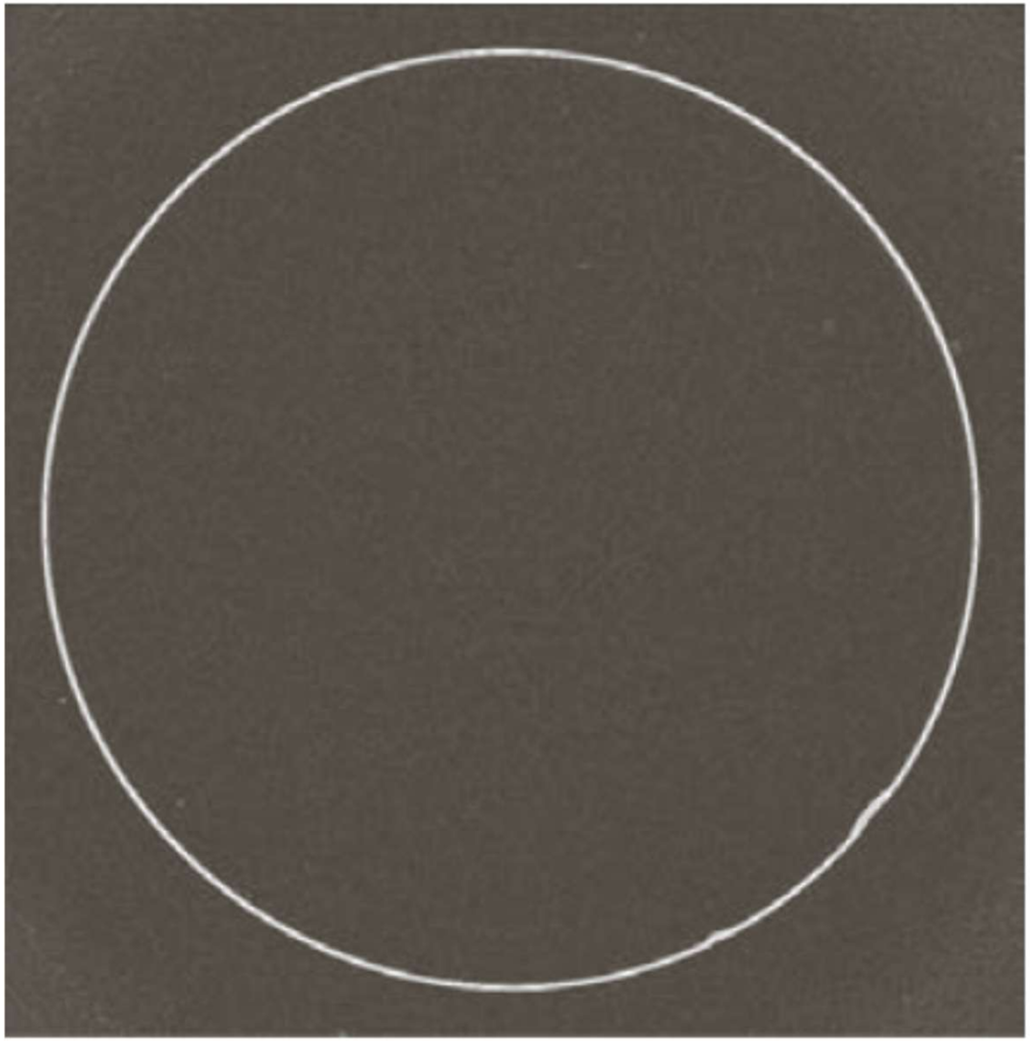
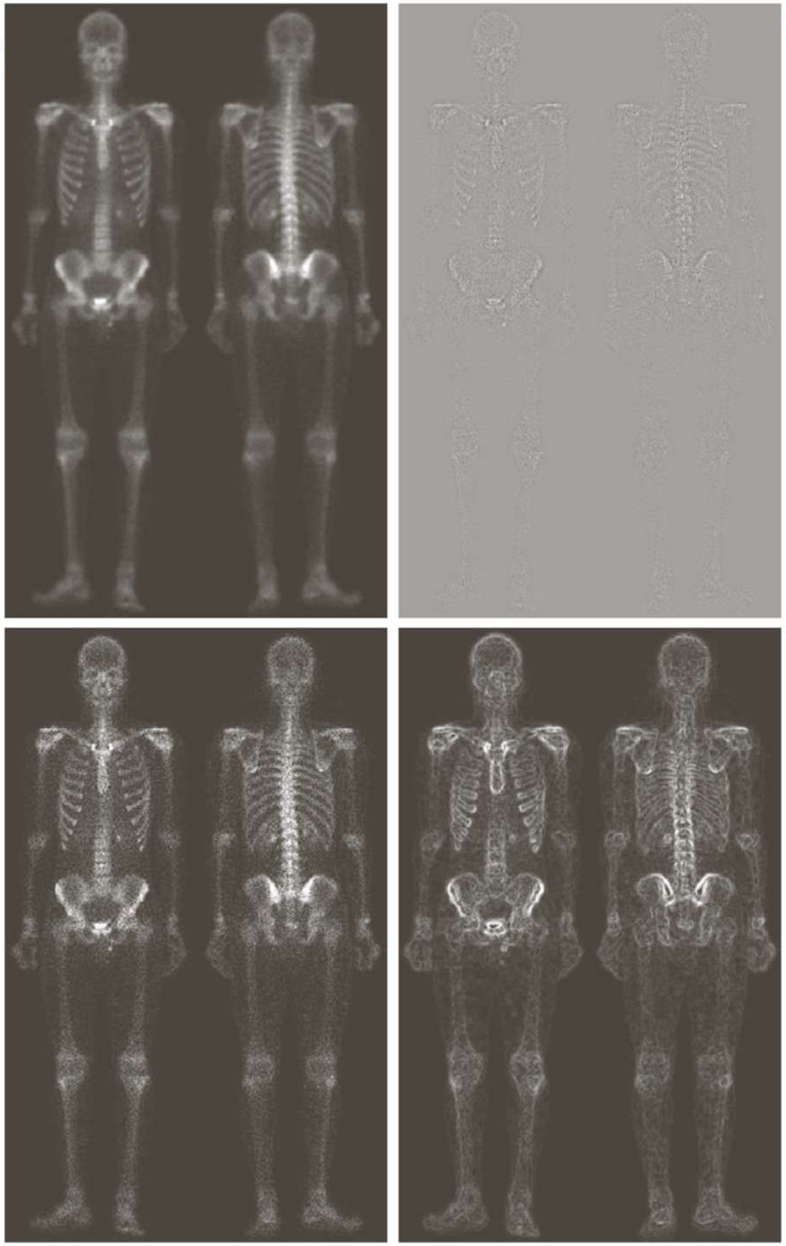


Image Enhancement In The Spatial Domain

- Combining spatial enhancement methods
- .

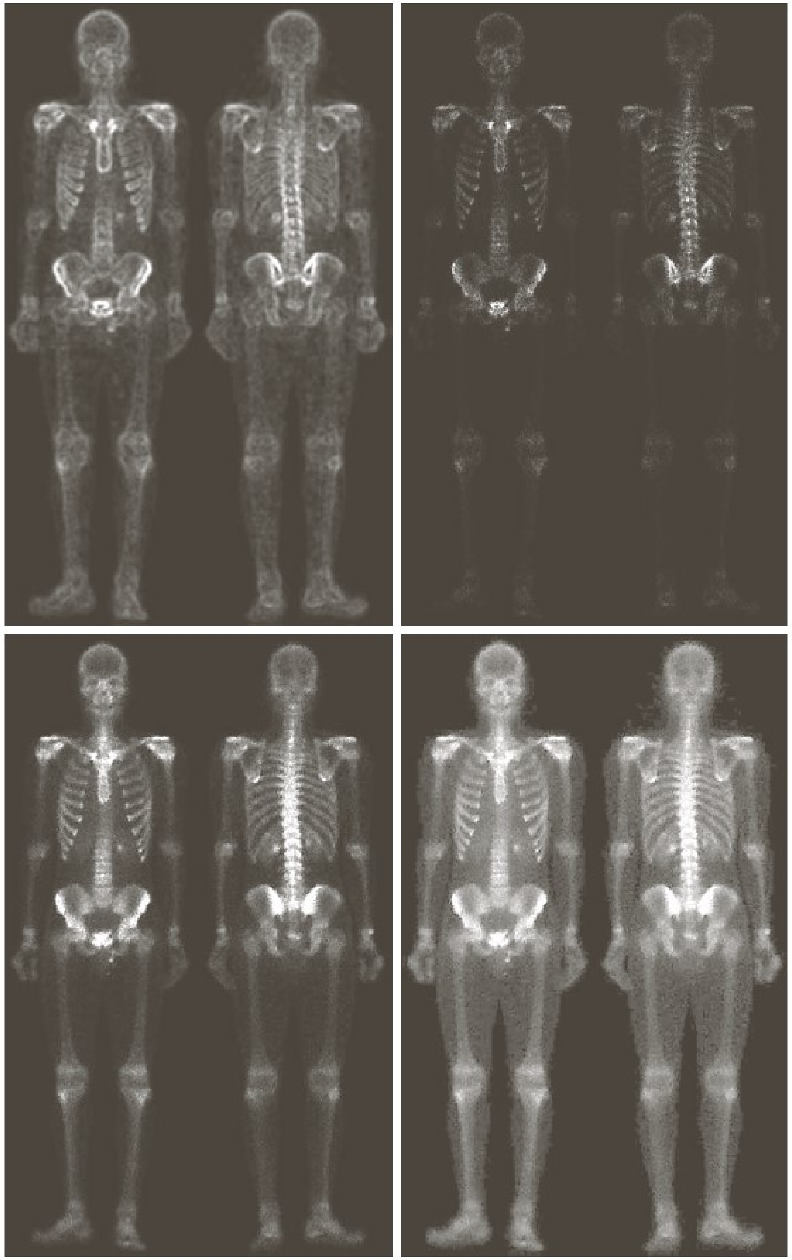
Ir The Spatial Domain



a	b
c	d

FIGURE 3.43
(a) Image of whole body bone scan.
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b).
(d) Sobel gradient of (a).

Ir The Sp ain



e f
g h

FIGURE 3.43
(Continued)
(e) Sobel image smoothed with a 5×5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

Image Enhancement In The Frequency Domain

- Module – 3 8 Hours
- Image Enhancement In Frequency Domain:
- Introduction,
- Fourier Transform,
- Discrete Fourier Transform (DFT),
- properties of DFT,
- Discrete Cosine Transform (DCT),
- Image filtering in frequency domain..

Some Basic Relationships Between Pixels